Trigonometric Identities Review

Part I
Prove the following identities.

1. $(1+\sin x)\cdot(\sec x-\tan x)=\cos x$

2. $\sin x\cdot(1+\cot^2 x)=\csc x$

3. $\csc^2 x-\cot^2 x=1$

4. $\tan x\cdot\sin^2 x\cdot\cos x=\sin^3 x$

5. $\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}=\sec x\cdot\csc x$

6. $\dfrac{1}{\cos x}-\cos x=\tan x\cdot\sin x$

7. $\sec x\cdot\tan x\cdot\csc x=\sec^2 x$

8. $(\cos x+\sin x)^2=1+2\cos x\cdot\sin x$

9. $\dfrac{\csc x\cdot\tan x}{\sec x}=1$

10. $\dfrac{1-\cos^4 x}{1+\cos^2 x}=\sin^2 x$

11. $\csc^4 x-\cot^4 x=\dfrac{1+\cos^2 x}{\sin^2 x}$

12. $(\tan x+\cot x)^2=\sec^2 x+\csc^2 x$

13. $\dfrac{1+\sin x}{1-\sin x}-\dfrac{1-\sin x}{1+\sin x}=4\tan x\cdot\sec x$

14. $\cos\left(\dfrac{\pi}{2}+x\right)=-\sin x$

15. $2\cot x\cdot\sin x=\csc x\cdot\sin(2x)$

16. $\cos(\pi+x)=-\cos x$

17. $\tan x+\cot x=2\csc(2x)$

18. $\sin(2x)\cdot\cos x-\cos(2x)\cdot\sin x=\sin x$

19. $\dfrac{\sin(2x)\cdot\cos x}{2}=\sin x-\sin^3 x$

20. $\sec^2 x\cdot\csc^2 x=\dfrac{4}{\sin^2(2x)}$

Part II

Use the sum and difference identities to find **exact** values for the following.

1. $\cos 75°$
2. $\sin 15°$
3. $\sin\dfrac{7\pi}{12}$
4. $\cos(-120°)$
5. $\tan\dfrac{13\pi}{12}$

Part III

Given that $\sin\alpha=\dfrac{4}{5}$ and $\cos\beta=\dfrac{12}{13}$, with α and β in quadrant I, find:

1. $\sin(\alpha+\beta)$
2. $\cos(\alpha+\beta)$
3. $\tan(\alpha+\beta)$

Given that $\cos\alpha=\dfrac{5}{13}$ and $\cos\beta=\dfrac{4}{5}$, with α and β in quadrant I, find:

4. $\sin(\alpha-\beta)$
5. $\cos(\alpha-\beta)$
6. $\tan(\alpha-\beta)$

Part IV

Solve for θ, where $0 \le \theta \le 2\pi$.

1. $\sin\theta + \sin(2\theta) = 0$
2. $\sin\theta = -\cos(2\theta)$
3. $\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = -\sin\theta$